Overview for today

- Recap: inference Lecture 2
- Presupposition revisited
- Definedness condition
- Pragmatic Theories

Reading:
- Coppock, E., and Champollion, L. (2021). Invitation to formal semantics. Manuscript, Boston University and New York University (Ch.8)
Simplify the following expression step-by-step

\[ \lambda Q. \forall x[\text{Linguist}(x) \to Q(x)](\lambda v_1. \text{Offended}(j, v_1)) \]
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\]

\[
\downarrow
\]

\[
\forall x [\text{Linguist}(x) \rightarrow \lambda v_1. \text{Offended}(j, v_1)(x)]
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\[ \Downarrow \]

\[ \forall x[Linguist(x) \rightarrow \text{Offended}(j, x)] \]
Recap: summary

Semantic Theories:

• contrast between assertion and presupposition (of an expression)
• presupposition as a different type of inference than logical implication or entailment
• presupposition as relation between sentences vs. statements (or even between speakers and statements)
• ambiguity of negation: structural (Russell) vs. lexical (Frege, Strawson)
Presupposition Projection (recap)

Presuppositions differ from semantic entailments because:

- presuppositions **survive** in contexts where entailments disappear (e.g. negation, modals, attitude verbs)
- presuppositions are **defeasible** e.g. they can disappear in contexts where entailments survive

⇒ Presupposition projection
Presupposition Defeasibility/Cancellation (recap)

**Contextual defeasibility**: the presupposition can be cancelled by the linguistic or non-linguistic context within the context of the same sentence or beyond the sentence, in the larger discourse context.

**Surface-structure defeasibility**: the presupposition is cancelled by a given surface-structure context (e.g. if-then, or) – *presupposition projection* problem
Presupposition Defeasibility/Cancellation (recap)

A presupposition can be cancelled by the linguistic or non-linguistic context within the context of the same sentence or beyond the sentence, in the larger discourse context.

1. When the linguistic context makes the presupposition inconsistent.
2. When is it common knowledge that the presupposition is false
3. When what is said, taken together with background assumptions makes the presupposition inconsistent.
4. When evidence for truth of the presupposition is being weighed and rejected

The projection problem has been dealt with using dynamic semantics, where the denotation of a sentence is a “context change potential”: a function that can update a discourse context.
Surface-Structure Defeasibility (recap)

There are cases of **intra-sentential cancellation or suspension** of presuppositions.

- A presupposition can “survive” i.e. project. We saw cases of this when the intra-sentential context contains *a negation, a modal, a disjunction and a conditional*.
- A presupposition can be “overtly cancelled or suspended” by the intra-sentential context.
- A presupposition can be “filtered” (i.e. partially let through) by intra-sentential contexts such as *and, if … then, but, suppose that*

John left work earlier again.

- John doesn’t regret leaving work early again because in fact he never did.
- John left work early again. What you mean again? He never did this before.
- John left work early again if in fact he ever did.
- John would leave work early again if he had a job.
- I don’t know whether John left work early again.
- John died before leaving work early again.
Semantic Presupposition: problems

**Problem 1** Presupposition failure (= the presupposition is false in context)

(1) King of France is bald.  
*When uttered on May 13 2005, the presupposition is false*

**Problem 2** Presupposition cancellation (= the presupposition is “removed” in context)

(2) If John has a wife, she (John’s wife) likes gardening.  
If John is married, his wife likes gardening.  
Either John got a divorce or his wife is helping him with work.  
\(\triangleright\) John has a wife.

Classical logic cannot handle presupposition failure; nor can it explain why sentences whose presuppositions are not satisfied are odd. To remedy this, semantic theories of presuppositions use multi-valued logics, which include true, false and neither-true-nor-false as possible truth-values.

Classical logic cannot account for the cancelling of presuppositions due to information available in the context. A possible remedy is to use a non-monotonic logic.
Moreover, many cases of what one would want to call presupposition are not truth-conditional effects, and are also strongly context-dependent. Therefore, the distinction between semantic and pragmatic presupposition is untenable and has been abandoned.

- Peter Frederick Strawson (1919-2006)
- Introduces an important distinction namely the distinction between sentences and use of sentences i.e. statements.
- Sentences aren’t true or false; Statements, i.e. ⟨Sentence, Context⟩ pairs, are.

Example

The King of France is wise.

- True in 1670.
- False in 1770.
- Neither true nor false in 1970.

Presuppositions are conventions about use of referring expressions: a statement A presupposes a statement B iff B is a precondition for the truth or falsity of A.
Pragmatic Theories of Presuppositions (recap)

Besides the (mostly abandoned) semantic attempts of modelling the projection problem, there are two main types of pragmatic theories:

(i) Theories based on a “static” semantics: Gazdar (1979), Karttunen (1973)


- Presuppositions are neither viewed as referring expressions nor as semantic entailments but as context-dependent (i.e. pragmatic) phenomena.
- When a presupposition conflicts with previous information, this presupposition
  - does not give rise to inconsistency
  - is lifted (i.e. cancelled) or altered (i.e. filtered) to resolve the conflict.
Presupposition Problems: summary

Problems with Semantic Theories:

- Cannot account for presupposition defeasibility.
  Proposed solution: Defeasibility is captured through binding or accommodation to a sub-level of the DRS (v.d. Sandt)
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Problems with Semantic Theories:

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Problems with Static Pragmatic Theories

- Semantic and presuppositional information are represented separately which yields wrong predictions concerning the communicated meaning. Proposed solution: Semantic and presuppositional information are represented in a uniform way. Problem does not occur (v.d. Sandt)
Presupposition Triggers

There are certain words/constructions that signal presupposition.

Example
Presupposition Triggers

There are certain words/constructions that signal presupposition.

Example

Every man kissed the woman who loved him. >>> There is at least one man and he kissed the woman who loved him.
Presupposition Triggers

There are certain words/constructions that signal presupposition.

Example

Every man kissed the woman who loved him. \(\Rightarrow\) There is at least one man and he kissed the woman who loved him.
Neither theory can explain why sentences whose presuppositions are not satisfied are odd. \(\Rightarrow\) There are exactly two theories. (quantifiers)
Presupposition Triggers

There are certain words/constructions that signal presupposition.

**Example**

*Every* man kissed the woman who loved him. $\gg\gg$ There is at least one man and he kissed the woman who loved him.

*Neither* theory can explain why sentences whose presuppositions are not satisfied are odd. $\gg\gg$ There are exactly two theories. (*quantifiers*)

John’s daughter will come. $\gg\gg$ John has a daughter. (*possessives*)
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John knows that Mary hates Bill. >>> Mary hates Bill (cognitive factive verbs).
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Mary bakes cookies again. >>> Mary has baked cookies before. (*additive adverbs*)
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*Every* man kissed the woman who loved him.  >>> There is at least one man and he kissed the woman who loved him.

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Mary bakes cookies *again*.  >>> Mary has baked cookies before.  (*additive adverbs*)

It was John *who* broke the computer.  >>> Someone broke the computer.  (*CLEFTS*)

The student is *smart*.  >>> There is an unique student in the context.  (*definite determiners*)
Definite Descriptions

Take predicates and return the unique individual, thus of type

Russell example

The princess smokes.

$\exists x. [\text{Princess}(x) \land \forall y. [\text{Princess}(y) \rightarrow x = y] \land \text{Smokes}(x)]$

Problems?

Strawson: EMPTY DESCRIPTIONS are definite descriptions in which nothing satisfies the descriptive content

Trivalent Strawsonian semantics: true, false and undefined
Definite Descriptions

Take predicates and return the unique individual, thus of type \( \langle\langle e, t \rangle, e \rangle \)

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Definite descriptions convey EXISTENCE and UNIQUENESS

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Trivalent Strawsonian semantics: true, false and undefined
Defined Descriptions: syntax and semantics

We introduce a special ‘undefined individual’ of type $e$ and use the symbol $#_e$ to denote this individual in our meta-language.

**Syntax Rule: Iota**

If $\phi$ is an expression of type $t$, and $u$ is a variable of type $e$, then $\iota u.\phi$ is an expression of type $e$.

**Semantic Rule: Iota**

$$\left[\iota u.\phi\right]_{M,g} = \begin{cases} d & \text{if } \{k|\left[\phi\right]_{M,g[u\rightarrow k]} = 1\} = \{d\} \\ #_e otherwise \end{cases}$$

\[the \rightsquigarrow \lambda P.\iota x.P(x)\]
Definite Descriptions (cont.)

Example

the president

\[
\text{DP} \\
\text{e} \\
\iota x. President(x) \\
\text{D} \quad \text{NP} \\
\langle \langle e, t \rangle, e \rangle \quad \langle e, t \rangle \\
\lambda P. \iota x. P(x) \quad \lambda x. President(x) \\
\text{the} \quad \text{NP} \\
\langle e, t \rangle \\
\lambda x. President(x) \\
\text{president}
\]
Definite Descriptions (cont.)

\[
\left[\forall x. \text{President}(x)\right]^{M,g} = \begin{cases} 
  d & \text{if } \{k|\left[\text{President}(x)\right]^{M,g[x\mapsto k]} = 1\} = \{d\} \\
  \#e & \text{otherwise}
\end{cases}
\]

why undefined value?

The King of France is bald.
The opera of Mozart is Italian.
Which of the structures lead to uninterpretability of a sentence like ‘Ann likes the book on the pillow’ and why?
Definateness Conditions

Definite determiner ‘the’ is one of the presupposition trigers, what about others? Both, neither, every, etc.
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Definite determiner ‘the’ is one of the presupposition triggers, what about others? Both, neither, every, etc.

$\partial$ operator is defined like $\partial(|P| = 2)$ reads ‘presupposing that there are exactly two Ps’.
Definiteness Conditions

Definite determiner ‘the’ is one of the presupposition triggers, what about others? Both, neither, every, etc.

∂ operator is defined like ∂(|P| = 2) reads ‘presupposing that there are exactly two Ps’.

neither ⇝ λP λQ.[∂(|P| = 2) ∧ ¬∃x.[P(x) ∧ Q(x)]]
Definateness Conditions

Definite determiner ‘the’ is one of the presupposition triggers, what about others? Both, neither, every, etc.

\( \partial \) operator is defined like \( \partial(|P| = 2) \) reads ‘presupposing that there are exactly two Ps’.

\[ \begin{align*}
neither & \leadsto \lambda P \lambda Q. [ \partial(|P| = 2) \land \neg \exists x. [P(x) \land Q(x)]] \\
every & \leadsto \lambda P \lambda Q. [ \partial(\exists x. P(x)) \land \forall x. [P(x) \rightarrow Q(x)]]
\end{align*} \]
Definiteness Conditions

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\]

Syntax Rule: Definedeness Conditions

If \( \phi \) is an expression of type \( t \), then \( \partial(\phi) \) is an expression of type \( t \)
Definiteness Conditions

Definite determiner ‘the’ is one of the presupposition triggers, what about others? Both, neither, every, etc.

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Syntax Rule: Definedeness Conditions

If \( \phi \) is an expression of type \( t \), then \( \partial(\phi) \) is an expression of type \( t \)

Semantic Rule: Definiteness Conditions

If \( \phi \) is an expression of type \( t \), then:

\[ [\partial(\phi)]^{M,g} = \begin{cases} 1 & \text{if } [\phi]^{M,g} = 1 \\ \#_e & \text{otherwise} \end{cases} \]
Definiteness Condition (cont.)

Example

Neither candidate is qualified.

\[ S \]

\[ \lambda x. \text{Qualified}(x) \]

is qualified

\[ \langle e, t \rangle \]

\[ \text{neither} \]

\[ \text{candidate} \]
Definiteness Condition (cont.)

Example

Neither candidate is qualified.

\[
S \\
\quad \text{DP} \\
\quad \quad \text{VP} \\
\quad \quad \quad \langle e, t \rangle \\
\quad \quad \quad \lambda x. \text{Qualified}(x) \\
\quad \quad \quad \quad \text{is qualified} \\
\quad \text{NP} \\
\quad \quad \langle e, t \rangle \\
\quad \quad \lambda x. \text{Candidate}(x) \\
\quad \quad \text{neither} \\
\quad \text{candidate}
\]
Definiteness Condition (cont.)

Example

Neither candidate is qualified.

\[
S = 
\begin{array}{c}
\lambda P \lambda Q. [\partial (|P| = 2) \land \neg \exists x. [P(x) \land Q(x)]] \\
\lambda x. Candidate(x)
\end{array}
\]

\[
\begin{array}{c}
\langle \langle e, t \rangle, e \rangle \\
\langle e, t \rangle
\end{array}
\]

\[
\lambda x. Qualified(x)
\]

is qualified

neither

candidate

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Defineness Condition (cont.)

Example

Neither candidate is qualified.

\[
S = \lambda Q. [\partial(|Candidate| = 2) \land \neg \exists x. [Candidate(x) \land Q(x)]]
\]

\[
\lambda P \lambda Q. [\partial(|P| = 2) \land \neg \exists x. [P(x) \land Q(x)]]
\]

is qualified

either

candidate
Definiteness Condition (cont.)

Example

Neither candidate is qualified.

\[ \partial(|\text{Candidate}| = 2) \land \neg \exists x.[\text{Candidate}(x) \land \text{Qualified}(x)] \]

Tree:

\[ \lambda Q. [\partial(|\text{Candidate}| = 2) \land \neg \exists x.[\text{Candidate}(x) \land Q(x)]] \]

\[ \lambda x. \text{Qualified}(x) \]

is qualified

\[ \lambda P \lambda Q. [\partial(|P| = 2) \land \neg \exists x.[P(x) \land Q(x)]] \]

\[ \lambda x. \text{Candidate}(x) \]

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candidate

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**Weak Kleene Connectives**

<table>
<thead>
<tr>
<th>∧</th>
<th>T</th>
<th>F</th>
<th>#</th>
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<tbody>
<tr>
<td>T</td>
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| ∨ | T | F | # | |
|---|---|---|---|
| T | T | T | # |
| F | T | F | # |
| # | # | # | # |

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|---|---|---|---|
| T | F | T | # |
| F | T | # | # |
| # | # | # | # |

**Idea:** We see as a # “contaminating” (or nonsense) value, which does not allow us to deduce anything if there is a presupposition failure somewhere. In Weak Kleene, any local presupposition failure leads to a global failure. If $[S] = #$, then any sentence that contains $S$ denotes $#$. 

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Idea: Idea: We see # in one argument as “ignorance” (unknown) - it still allows us to deduce the result from the value of the other argument.
Universal & Existential Quantifiers

Every boy loves his cat.

\( \forall x. [\text{Boy}(x) \rightarrow \text{Loves}(x, y).[\text{Cat}(y) \land \text{Has}(x, y)]] \) (UNIVERSAL PROPOSITION)

every element of \( D_e \) is \( [\text{Boy}(x) \rightarrow \text{Loves}(x, y).[\text{Cat}(y) \land \text{Has}(x, y)]] \)

\( x \) loves his cat (SCOPE PROPOSITION) where \( x \) is 1 or \# should UNIVERSAL PROPOSITION be 1 or \#?

Muskens (1995) sees universal claim as a big conjunction

\[
[\forall x. \phi]^{M,g} = \begin{cases} 
1 & \text{if } [\phi]^{M,g[x \mapsto k]} = 1 \text{ for all } k \in D \\
\# & \text{if } [\phi]^{M,g[x \mapsto k]} = \# \text{ for some } k \in D \\
0 & \text{otherwise}
\end{cases}
\]

and existential claim as big disjunction

\[
[\exists x. \phi]^{M,g} = \begin{cases} 
0 & \text{if } [\phi]^{M,g[x \mapsto k]} = 0 \text{ for all } k \in D \\
\# & \text{if } [\phi]^{M,g[x \mapsto k]} = \# \text{ for some } k \in D \\
1 & \text{otherwise}
\end{cases}
\]
Identity

The King of France is the Grand Sultan of Germany.

LaPierre (1992) defines identity between two terms as follows:

- if neither $\alpha$ nor $\beta$ denotes the undefined individual, then $\llbracket \alpha = \beta \rrbracket^{M,g} = 1$ if $\llbracket \alpha \rrbracket^M, g = \llbracket \beta \rrbracket^M, g$, and 0 otherwise.
- If one $\alpha$ or $\beta$ denotes the undefined individual, then $\llbracket \alpha = \beta \rrbracket^{M,g} = 0$
- If both denote the undefined individual, then $\llbracket \alpha = \beta \rrbracket^{M,g} = \#$ is undefined (not enough is “known” about the objects to determine that they are the same or distinct).
Predication with Undefined Individuals

Semantic Rule: Existence Predicate

\[ \langle \text{Exists}(\alpha) \rangle^{M,g} = 1 \text{ if } \langle \alpha \rangle^{M,g} \neq \#_e \text{ and 0 otherwise} \]

Predict truth value of ‘The Golden Mountain does not exist’
Semantics of $\partial L$

Type $e$ is associated with the domain of individuals $D_e = D$
Semantics of $\mathcal{L}$

Type $e$ is associated with the domain of individuals $D_e = D$
Type $t$ is associated with the domain of truth values $D_t = \{1, 0, \#\}$
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For functional types $\langle \sigma, \tau \rangle$ there is a domain $D_{\langle \sigma, \tau \rangle}$ consisting of the (total) functions from $D_\sigma$ to $D_\tau$. 
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For every type, there is also an ‘undefined individual’ of that type, which we refer to $\#_e$. 
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- The domain of individuals $D_e$ contains at least one individual.
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- The domain of individuals $D_e$ contains at least one individual.
- $I$ is an interpretation function, assigning a denotation to all of the constants of the language. The denotation of a constant of type $\tau$ is a member of $D_\tau$. 
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An assignment $g$ is a total function whose domain consists of the variables of the language such that if $u$ is a variable of type $\tau$ then $g(u) \in D_\tau$. 


Semantics of $\partial L$

Type $e$ is associated with the domain of individuals $D_e = D$  
Type $t$ is associated with the domain of truth values $D_t = \{1, 0, \#\}$  
For functional types $\langle \sigma, \tau \rangle$ there is a domain $D_{\langle \sigma, \tau \rangle}$ consisting of the (total) functions from $D_\sigma$ to $D_\tau$.  
For every type, there is also an ‘undefined individual’ of that type, which we refer to $\#_e$.  
  
- The domain of individuals $D_e$ contains at least one individual.  
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An assignment $g$ is a total function whose domain consists of the variables of the language such that if $u$ is a variable of type $\tau$ then $g(u) \in D_\tau$.  
We use $g[x \mapsto d]$ to denote an assignment function which is exactly like $g$ with the possible exception that $g(x) = d$.  

Quizz for Today

TBA