Introduction to Formal Semantics
Tutorial Lecture 8: Intensional and Modal Logic

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Overview for today

1. Coextensionality
2. Informative/Uninformative
3. Towards Intensionality
4. Contingent and Necessary Truth
5. Propositional Attitudes
6. Intension and composition

Reading:
1. Coextensionality
1. Coextensionality

**Substitutability of coextensionals**

> If two expressions\(^1\) have the same extension, then if one is substituted for the other in any given sentence, the truth value of the sentence remains the same. *(Coppock & Champollion 2022, p. 489)*

1. Usually, with reference to Proper nouns (PNs), Definite descriptions (NPs). Still, every expression\(_X\) : expression\(_A\) \(\models\) expression\(_B\)
1. Coextensionality

Substitutability of coextensionals

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\[[\text{Peter Parker}]\]

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1. Coextensionality

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If two expressions have the same extension, then if one is substituted for the other in any given sentence, the truth value of the sentence remains the same. (Coppock & Champollion 2022, p. 489)

a. Peter Parker kisses Mary Jane.

1. Usually, with reference to Proper nouns (PNs), Definite descriptions (NPs). Still, every expression_\text{A} : \text{expression}_\text{A} := \text{expression}_\text{B}
1. Coextensionality

**Substitutability of coextensionals**

If two expressions\(^1\) have the same extension, then if one is substituted for the other in any given sentence, the truth value of the sentence remains the same. (Coppock & Champollion 2022, p. 489)

a. Peter Parker kisses Mary Jane.

b. Peter Parker is Spiderman

---

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1. Coextensionality

**Substitutability of coextensionals**

If two expressions\(^1\) have the same extension, then if one is substituted for the other in any given sentence, the truth value of the sentence remains the same. *(Coppock & Champollion 2022, p. 489)*

a. **Peter Parker** kisses Mary Jane.

b. **Peter Parker** is **Spiderman**

c. \(\therefore\) **Spiderman** kisses Mary Jane.

---

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If two expressions\(^1\) have the same extension, then if one is substituted for the other in any given sentence, the truth value of the sentence remains the same. \(\text{(Coppock & Champollion 2022, p. 489)}\)

1. Usually, with reference to Proper nouns (PNs), Definite descriptions (NPs). Still, every expression\(_X\) : expression\(_A\) \(\models\) expression\(_B\)

\begin{itemize}
  \item a. Peter Parker kisses Mary Jane.
  \item b. Peter Parker is Spiderman
  \item c. \(\therefore\) Spiderman kisses Mary Jane.
\end{itemize}
1. Coextensionality

Substitutability of coextensionals

If two expressions have the same extension, then if one is substituted for the other in any given sentence, the truth value of the sentence remains the same. (Coppock & Champollion 2022, p. 489)

a. Peter Parker kisses Mary Jane.
b. Peter Parker is Spiderman
c. ∴ Spiderman kisses Mary Jane.

= ✓

1. Usually, with reference to Proper nouns (PNs), Definite descriptions (NPs). Still, every expression : expression_\lambda \models expression_\beta
1. Coextensionality

Substitutability of coextensionals

If two expressions\(^1\) have the same extension, then if one is substituted for the other in any given sentence, the truth value of the sentence remains the same. (Coppock & Champollion 2022, p. 489)

\[ M_{De} = \{e_1, e_2, e_3\} \]
\[ I_M(peter \ parker) = I_M(spiderman) = e_1 \]
\[ I_M(mary \ jane) = e_2 \]
\[ I_M(harry \ osborne) = e_3 \]

---

1. Usually, with reference to Proper nouns (PNs), Definite descriptions (NPs). Still, every expression\(_X\) : expression\(_A\) ⊨ expression\(_B\)
If two expressions have the same extension, then if one is substituted for the other in any given sentence, the truth value of the sentence remains the same. *(Coppock & Champollion 2022, p. 489)*

1. Usually, with reference to Proper nouns (PNs), Definite descriptions (NPs). Still, every expression \( X \): expression \( A \models \) expression \( B \)
1. Coextensionality

**Substitutability of coextensionals**

If two expressions\(^1\) have the same extension, then if one is substituted for the other in any given sentence, the truth value of the sentence remains the same. *(Coppock & Champollion 2022, p. 489)*

\[
M_{De} = \{e_1, e_2, e_3\}
\]

\[
I_M(\text{peter parker}) = I_M(\text{spiderman}) = e_1
\]

\[
I_M(\text{mary jane}) = e_2
\]

\[
I_M(\text{harry osborne}) = e_3
\]
If “a” and “b” are co-referential expressions then by substituting one with the other “φ[^a/b^]” they are co-extensional: $[[φ[^a^]]] = T \iff [[φ[^b^]]] = T$

2. Here co-referentiality does not subsume deictic terms/anaphors as they first need to be anchored within the context.
2. Informative/Uninformative
## Informative Vs Uninformative

**Keep in mind!** Co-extensionality works because of \( a = b \) and not \( a = a \) (tautological)
2. Informative/Uninformative

Keep in mind! Co-extensionality works because of $a = b$ and not $a = a$ (tautological)

- Informative $(a = b)$
- Uninformative $(a = a)$
Informative Vs Uninformative

Keep in mind! Co-extensionality works because of $a = b$ and not $a = a$ (tautological)

- Informative $(a = b)$
  - (1a) J.K. Rowling is the author of Harry Potter.
Informative Vs Uninformative

Keep in mind! Co-extensionality works because of \( a = b \) and not \( a = a \) (tautological)

▶ Informative \((a = b)\)

(1a) J.K. Rowling is the author of Harry Potter.
(2a) The empire state building is the 4th tallest building in New York City.
Informative Vs Uninformative

Keep in mind! Co-extensionality works because of \( a = b \) and not \( a = a \) (tautological)

- **Informative** \( (a = b) \)
  1. \( J.K.\) Rowling is the author of Harry Potter.
  2. The empire state building is the 4th tallest building in New York City.

- **Uninformative** \( (a = a) \)
Informative Vs Uninformative

Keep in mind! Co-extensionality works because of $a = b$ and not $a = a$ (tautological)

▶ Informative $(a = b)$

(1a) J.K. Rowling is the author of Harry Potter.
(2a) The empire state building is the 4th tallest building in New York City.

▶ Uninformative $(a = a)$

(3a) Susan is Susan
2. Informative/Uninformative

Informative Vs Uninformative

Keep in mind! Co-extensionality works because of $a = b$ and not $a = a$ (tautological)

- **Informative** ($a = b$)
  1. (1a) J.K. Rowling is the author of Harry Potter.
  2. (2a) The empire state building is the 4th tallest building in New York City.

- **Uninformative** ($a = a$)
  3. (3a) Susan is Susan
  4. (4a) Boys are boys

3. Semantically uninformative, but pragmatically it may convey some sort of information (e.g. Speaker flouts the Maxim of Quantity.)
Informative Vs Uninformative

Keep in mind! Co-extensionality works because of $a = b$ and not $a = a$ (tautological)

- **Informative** ($a = b$)
  1. (1a) J.K. Rowling is the author of Harry Potter.
  2. (2a) The empire state building is the 4th tallest building in New York City.

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Informative Vs Uninformative

Keep in mind! Co-extensionality works because of $a = b$ and not $a = a$ (tautological). This is also evident when the identity relation is negated.

- Informative $(a = b)$, under negation $(a = \lnot b)$

- Uninformative
  - $(3b)$ Susan is NOT Susan
  - $(4b)$ Boys are NOT boys
Informative Vs Uninformative

Keep in mind! Co-extensionality works because of $a = b$ and not $a = a$ (tautological). This is also evident when the identity relation is negated.

Informative $(a = b)$, under negation $(a = \neg b)$

Note: under negation $a = \neg b$ is still informative! This renders it contingent in nature and will allow us to consider modal logic and possible worlds.
Informative Vs Uninformative

Keep in mind! Co-extensionality works because of \( a = b \) and not \( a = a \) (tautological). This is also evident when the identity relation is negated.

- Informative \((a = b)\), under negation \((a = \neg b)\)

  (1b) J.K. Rowling is NOT the author of Harry Potter\(^4\).

---

4. We might learn J.K. Rowling had a ghostwriter.
Informative Vs Uninformative

Keep in mind! Co-extensionality works because of \( a = b \) and not \( a = a \) (tautological). This is also evident when the identity relation is negated.

- Informative \((a = b)\), under negation \((a = \neg b)\)
  
  (1b) J.K. Rowling is NOT the author of Harry Potter\(^4\).
  
  (2b) The empire state building is NOT the 4th tallest building in New York City\(^5\).

---

4. We might learn J.K. Rowling had a ghostwriter.
5. Eventually, the building might become the 5th tallest building in NYC.
Informative Vs Uninformative

Keep in mind! Co-extensionality works because of $a = b$ and not $a = a$ (tautological). This is also evident when the identity relation is negated.

▶ Informative $(a = b)$, under negation $(a = \neg b)$ ✓

(1b) J.K. Rowling is NOT the author of Harry Potter.
(2b) The empire state building is NOT the 4th tallest building in New York City.

4. We might learn J.K. Rowling had a ghostwriter.
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Keep in mind! Co-extensionality works because of $a = b$ and not $a = a$ (tautological). This is also evident when the identity relation is negated.

- **Informative** ($a = b$), under negation ($a = \neg b$) ✓
  
  1b) J.K. Rowling is NOT the author of Harry Potter⁴.
  
  2b) The empire state building is NOT the 4th tallest building in New York City⁵.

- **Uninformative** ($a = a$), under negation ($a = \neg a$)

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Informative Vs Uninformative

- **Informative** ($a = b$), under negation ($a = \lnot b$)
  - (1b) J.K. Rowling is NOT the author of Harry Potter.
  - (2b) The empire state building is NOT the 4th tallest building in New York City.

- **Uninformative** ($a = a$), under negation ($a = \lnot a$)
  - (3b) Susan is NOT Susan

---

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Informative Vs Uninformative

Keep in mind! Co-extensionality works because of $a = b$ and not $a = a$ (tautological). This is also evident when the identity relation is negated.

- **Informative** $(a = b)$, **under negation** $(a = \neg b)$ ✓
  1b) J.K. Rowling is NOT the author of Harry Potter.
  2b) The empire state building is NOT the 4th tallest building in New York City.

- **Uninformative** $(a = a)$, **under negation** $(a = \neg a)$
  3b) Susan is NOT Susan
  4b) Boys are NOT boys

4. We might learn J.K. Rowling had a ghostwriter.
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Keep in mind! Co-extensionality works because of $a = b$ and not $a = a$ (tautological). This is also evident when the identity relation is negated.

- Informative $(a = b)$, under negation $(a = \neg b)$ ✅
  
  (1b) J.K. Rowling is NOT the author of Harry Potter.
  
  (2b) The empire state building is NOT the 4th tallest building in New York City.

- Uninformative $(a = a)$, under negation $(a = \neg a)$ ✗
  
  (3b) Susan is NOT Susan
  
  (4b) Boys are NOT boys

4. We might learn J.K. Rowling had a ghostwriter.
5. Eventually, the building might become the 5th tallest building in NYC.
Is the principle of co-extensionality always effective?
3. Towards Intensionality
3. Towards Intensionality

The problem of substitutivity of co-referential terms

(5) The empire state building is the 4th tallest building in NYC.

Intension ↓[esb]

- $a = \text{the tallest building in NYC.}$
- $b = \text{the 2nd tallest building in NYC.}$
- $c = \text{the 3rd tallest building in NYC.}$
- $d = \text{the 4th tallest building in NYC.}$
- $e = \text{the 424 ft building located in e*}$

Extension [esb]

*e ...the 424 ft building located in 20 W 34th St, NT, 1001, U.S.
3. Towards Intensionality

The problem of substitutivity of co-referential terms

(5) The empire state building is the 4th tallest building in NYC (Contingent truth)

▶︎ a = the tallest building in NYC.
▶︎ b = the 2nd tallest building in NYC.
▶︎ c = the 3rd tallest building in NYC.
▶︎ d = the 4th tallest building in NYC.
▶︎ e = the 424 ft building located in e*

*e ...the 424 ft building located in 20 W 34th St, NT, 1001, U.S. Is “e” necessary or contingent? Moreover, what about if our Model
(5) The empire state building is the 4th tallest building in NYC (Contingent truth)

- **a** = the tallest building in NYC.
- **b** = the 2nd tallest building in NYC.
- **c** = the 3rd tallest building in NYC.
- **d** = the 4th tallest building in NYC.
- **e** = the 424 ft building located in 20 W 34th St, NT, 1001, U.S. Is “e” necessary or contingent? Moreover, what about if our Model...
What is the causes this?
4. Contingent and Necessary Truth
“Whether or not a proposition necessarily holds depends on its truth value in every world, not just the world under consideration. In other words, necessarily depends on the INTENSION of the sentence it combines with, and not just its EXTENSION. The extension of an expression is its semantic value at a particular world (so, for formulas, the extension is a truth value), while the intension is a function from possible worlds to the extensions they have at those worlds.” (p. 491)
A necessary truth is one that could not have been otherwise. In all circumstances, a necessary truth expresses a true proposition.

(6) Human beings are mortal.

...is the sentence true or false?
Necessary truth

A **necessary truth** is one that could not have been otherwise. In **all circumstances**, a necessary truth expresses a true proposition.

(6) Human beings are mortal.

(i) **Necessarily**, human beings are mortal.

\[
\begin{array}{c}
\text{Alethic} \\
\square \\
\phi \\
\text{...is true in every-possible situation}
\end{array}
\]
4. Contingent and Necessary Truth

Contingent truth

A **contingent truth** is one that is true, but could have been false. In **some circumstances**, a contingent truth expresses a true proposition.

(7) The empire state building is the 4th tallest building in NYC.

...is the sentence true or false?
A **contingent truth** is one that is true, but could have been false. In **some circumstances**, a contingent truth expresses a true proposition.

(7) The empire state building is the 4th tallest building in NYC.

(ii) **Possibly**, the empire state building is ...

\[
\Diamond \phi \quad \text{...is true in some-possible situations}
\]
Intensional operators

... we call these “circumstances” worlds $w \in W$ — Leibniz first defined possible worlds, Kripke later formalized this idea — and are now able to extend our traditional model into an intensional $\langle D, W, I \rangle$. The following are the two basic new intensional operators: “Box operator $\Box$”, “Diamond operator $\Diamond$”.

- If $\phi$ is a formula, then $\Box\phi$ is a formula.
- If $\phi$ is a formula, then $\Diamond\phi$ is a formula.
... we call these “circumstances” worlds \( w \in W \) — Leibniz first defined possible worlds, Kripke later formalized this idea — and are now able to extend our traditional model into an intensional \( \langle D, W, I \rangle \). The following are the two basic new intensional operators: “Box operator \( \Box \)”, “Diamond operator \( \Diamond \).

- If \( \phi \) is a formula, then \( \Box \phi \) is a formula.
- If \( \phi \) is a formula, then \( \Diamond \phi \) is a formula.

Truth conditions

- \[ [\Box \phi]_{M,g,w} = T \text{ iff } [\phi]_{M,g,w'} = T \text{ for all } w' \]
- \[ [\Diamond \phi]_{M,g,w} = T \text{ iff } [\phi]_{M,g,w'} = T \text{ for some } w' \]
4. Contingent and Necessary Truth

### Reflexitivity of the intensional operators

**Reflexivity of □:**

If □φ is true in all worlds (w∈W), then φ is also true in the actual world (w@):

▶ □φ → φ

**Non Reflexivity of ◊:**

If ◊φ is true in some worlds, then φ is not necessarily true in the actual world:

▶ ◊φ → φ, however, the following holds: φ → ◊φ

□φ ←→ ¬◊¬φ are equivalent
Expressions based on their intentionality are interpreted as follow:

- **Proper name:**
  \[
  \llbracket a_e \rrbracket_{M,g,w} = \llbracket \{ w_0 \mapsto a, \ldots, w_n \mapsto a \} \rrbracket \quad \text{(rigid designators)}
  \]

- **Predicates:**
  \[
  \llbracket P_{et} \rrbracket_{M,g,w} = \llbracket \{ w_0 \mapsto \{ a, b \}, \ldots, w_n \mapsto \{ a, b, d \} \} \rrbracket
  \]

- **Sentences:**
  \[
  \llbracket \phi_t \rrbracket_{M,g,w} = \llbracket \{ w_0 \mapsto 1, \ldots, w_n \mapsto 0 \} \rrbracket
  \]
Expressions based on their intentionality are interpreted as follow:

- **Proper name:**
  \[ I(w_3, \text{john}) = \text{john} \]

- **Predicates:**
  \[ I(w_3, \text{Happy}) = \{\text{john, sabrina, lela}\} \]

- **Sentences:**
  \[ [\text{Happy(john)}]_{M,g,w_3} = 1 \]
Now we can move from the extensional reading...
(8) ... is Spiderman.

\[
\begin{align*}
a. \quad (\llbracket M, g \rrbracket \downarrow M, g = \llbracket \text{Spiderman} \rrbracket \downarrow M, g) &= ? \\
b. \quad (\llbracket M, g \rrbracket \downarrow M, g = \llbracket \text{Spiderman} \rrbracket \downarrow M, g) &= ? \\
c. \quad (\llbracket M, g \rrbracket \downarrow M, g = \llbracket \text{Spiderman} \rrbracket \downarrow M, g) &= ? \\
d. \quad (\llbracket M, g \rrbracket \downarrow M, g = \llbracket \text{Spiderman} \rrbracket \downarrow M, g) &= ? \\
\end{align*}
\]

...and which wouldn’t be that helpful.
4. Contingent and Necessary Truth

To an intensional reading…
(8) ... is Spiderman.

a. \( \llbracket \text{John} \rrbracket^{M,g} = \llbracket \text{Spiderman} \rrbracket^{M,g}(w?) = ? \)

b. \( \llbracket \text{Peter} \rrbracket^{M,g} = \llbracket \text{Spiderman} \rrbracket^{M,g}(w?) = ? \)

c. \( \llbracket \text{Tom} \rrbracket^{M,g} = \llbracket \text{Spiderman} \rrbracket^{M,g}(w?) = ? \)

d. \( \llbracket \text{Denzel} \rrbracket^{M,g} = \llbracket \text{Spiderman} \rrbracket^{M,g}(w?) = ? \)

...where, given our possible worlds, one interpretation holds rather than the other.
4. Contingent and Necessary Truth

(8) ... is Spiderman.

a. \( (\llbracket \text{\textbullet} \rrbracket_{\mathcal{M},g} = \llbracket \text{\spider} \rrbracket_{\mathcal{M},g})(w_1) = ? \)

b. \( (\llbracket \text{\textbullet} \rrbracket_{\mathcal{M},g} = \llbracket \text{\spider} \rrbracket_{\mathcal{M},g})(w_2) = ? \)

c. \( (\llbracket \text{\textbullet} \rrbracket_{\mathcal{M},g} = \llbracket \text{\spider} \rrbracket_{\mathcal{M},g})(w_3) = ? \)

d. \( (\llbracket \text{\textbullet} \rrbracket_{\mathcal{M},g} = \llbracket \text{\spider} \rrbracket_{\mathcal{M},g})(w_4) = ? \)

\( w_1 = \text{marvel comics}, w_2 = \text{marvel Sony universe}, w_3 = \text{marvel sony universe}, w_4 = \text{marvel cinematic universe} \)
4. Contingent and Necessary Truth

(8) ... is Spiderman.

a. \( \langle \text{man} \rangle_{M,g} = \langle \text{Spiderman} \rangle_{M,g}(w_1) = ? \)

b. \( \langle \text{man} \rangle_{M,g} = \langle \text{Spiderman} \rangle_{M,g}(w_2) = ? \)

c. \( \langle \text{man} \rangle_{M,g} = \langle \text{Spiderman} \rangle_{M,g}(w_3) = ? \)

d. \( \langle \text{man} \rangle_{M,g} = \langle \text{Spiderman} \rangle_{M,g}(w_4) = ? \)

w_1 = marvel comics, w_2 = marvel Sony universe, w_3 = marvel sony universe, w_4 marvel cinematic universe
4. Contingent and Necessary Truth

\[
\begin{align*}
(8) & \quad \downarrow[a] & \quad \downarrow[b] & \quad \downarrow[c] & \quad \downarrow[d] \\
& \begin{bmatrix}
  w_1 & \rightarrow & 1 \\
  w_2 & \rightarrow & 1 \\
  w_3 & \rightarrow & 1 \\
  w_@ & \rightarrow & 1 \\
\end{bmatrix} & \begin{bmatrix}
  w_1 & \rightarrow & 0 \\
  w_2 & \rightarrow & 1 \\
  w_3 & \rightarrow & 0 \\
  w_@ & \rightarrow & 1 \\
\end{bmatrix} & \begin{bmatrix}
  w_1 & \rightarrow & 0 \\
  w_2 & \rightarrow & 1 \\
  w_3 & \rightarrow & 0 \\
  w_@ & \rightarrow & 1 \\
\end{bmatrix} & \begin{bmatrix}
  w_1 & \rightarrow & 0 \\
  w_2 & \rightarrow & 1 \\
  w_3 & \rightarrow & 1 \\
  w_@ & \rightarrow & 1 \\
\end{bmatrix}
\end{align*}
\]

\(w_1 = \) marvel comics, \(w_2 = \) marvel Sony universe, \(w_3 = \) marvel sony universe, \(w_4 = \) marvel cinematic universe
4. Contingent and Necessary Truth

\[ W_0 \]
\[ W_1 \]
\[ W_2 \]
\[ W_3 \]
4. Contingent and Necessary Truth

(8) a. $M_{\{w@, w1, w2, w3\}} \models \text{Peter Parker is Spiderman}$
b. $M_{\{w@, w2\}} \models \text{Toby Maguire is Spiderman}$
c. $M_{\{w@, w2\}} \models \text{Andrew Garfield is Spiderman}$
d. $M_{\{w@, w3\}} \models \text{Tom Holland is Spiderman}$
4. Contingent and Necessary Truth

(8)  

a. $M_{\{w@, w_1, w_2, w_3\}} \models \Box \text{Peter Parker is Spiderman}$  
b. $M_{\{w@, w_2\}} \models \Diamond \text{Toby Maguire is Spiderman}$  
c. $M_{\{w@, w_2\}} \models \Diamond \text{Andrew Garfield is Spiderman}$  
d. $M_{\{w@, w_3\}} \models \Diamond \text{Tom Holland is Spiderman}$
5. Propositional attitudes
5. Propositional Attitudes

### Propositional attitude verbs

These express the speaker's attitude towards a certain proposition. e.g. *believe, know, want*. In particular, **belief states** are crucial to reason about the **common ground** of two or more speakers.
Propositional attitude verbs

These express the speaker's attitude towards a certain proposition. e.g. believe, know, want. In particular, belief states are crucial to reason about the common ground of two or more speakers.

(9) Susan believes Peter Parker is Iron Man.
5. Propositional Attitudes

Propositional attitude verbs

These express the speaker's attitude towards a certain proposition. e.g. believe, know, want. In particular, belief states are crucial to reason about the common ground of two or more speakers.

(9) Susan \textbf{believes} Peter Parker is Iron Man.

\textbf{Dox} \quad \phi

(9) Susan \textbf{believes} Peter Parker is Iron Man. \neq \phi
Belief states and the common ground

These express the speaker's attitude towards a certain proposition. e.g. believe, know, want. In particular, belief states are crucial to reason about the common ground of two or more speakers.

(10) \([\text{Peter Parker is Spider Man}]^{M,g} = \begin{bmatrix}
  w_{mj} & \mapsto & 1 \\
  w_{ned} & \mapsto & 1 \\
  w_{oct} & \mapsto & 1 \\
  w_{sus} & \mapsto & 0
\end{bmatrix}\)
2.2 Pre-tests and Item conditions

**Pre-Test 2**
Access availability of relevant beliefs of the participants, testing for **common ground**. e.g. if the participants failed to guess the right country the sentence got **discarded** and substituted by a sentence that passed the pre-test.

**Common Ground**

**Cloze Test**
1. The fall of the Berlin Wall reunited _______
2. The .............................................. _______
3. The .............................................. _______

\[ \lambda x. [\text{The fall of the Berlin Wall reunited } (x)] (\text{Germany})^{\text{COUNTRY}} \]

Nicolae Dominik Dascalu: Integration of Word Meaning and World Knowledge in Language Comprehension
Veridicals

However, not all propositional attitude verbs give an intensional reading of their complement.

Note: remember factive verbs?

(11) Tidus knows that \( \phi \).

\[ \Rightarrow \ \phi \text{ is the case; } \models \phi \]

(12) Auron noticed that \( \phi \).

\[ \Rightarrow \ \phi \text{ is the case; } \models \phi \]
These two readings are called **DE RE** ('of the object') and **DE DICTO** ('of the word'). In the **de re reading**, Andrew noticed a specific job offer. According to the **de dicto reading**, Andrew desires a job offer in general.

(13) Andrew *saw a job offer*. **De Re**

(14) Andrew *wants a job offer*. 
## 5. Propositional Attitudes

### De Re vs. De Dicto Reading

These two readings are called **DE RE** ('of the object') and **DE DICTO** ('of the word'). In the **de re reading**, Andrew noticed a specific job offer. According to the **de dicto reading**, Andrew desires a job offer in general.

(13) Andrew saw a job offer.

(14) Andrew **wants** a job offer.

---

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Introduction to Formal Semantics, Summer 2022

20.06.22
The *de re/de dicto* distinction is related to the distinction between specific and nonspecific objects. In many languages, indefinites can be marked for specificity using what is known as **Differential Object Marking (DOM)**. (p. 493)

(13) Andrew *saw* a job offer.

(14) Andrew *wants* a job offer.
The *de re/de dicto* distinction is related to the distinction between specific and nonspecific objects. In many languages, indefinites can be marked for specificity using what is known as **Differential Object Marking (DOM)**. (p. 493)

(15) a. Juan busca **a un profesor**.
    b. Juan busca **un profesor**.
How to treat belief states?
On the *de re* reading, Ralph has a belief about a particular object/individual: There is someone about whom Ralph believes that they are a spy. *(see Quine 1956)*

\[(16) \text{a. Ralph believes that *someone* is a spy} \quad \text{De Re} \]

\[(b. \exists x [\text{Bel}(\text{ralph, Spy}(x))] \quad \text{DoxScope}] \]
On the *de dicto* reading, Ralph has no particular individual in mind; he just believes that there are spies. The belief is not about a particular individual, rather it’s about the category, spies.

(16) a. Ralph believes that *someone is a spy*  

\[
\text{c. } \text{Bel}(\text{ralph}, \exists x [\text{Spy}(x)])]
\]

6. Among all e ∈ D, some are spies.
## Opacity vs. Transparency

If the context of speech is clear we can substitute co-referential terms and implement them into reasoning patterns. However, **propositional attitude verbs** — in particular the verbs *believe* and *know* — give rise to environments where the principle of co-extensionality might fail. These are called **opaque**.
If the context of speech is clear we can substitute co-referential terms and implement them into reasoning patterns. However, propositional attitude verbs— in particular the verbs believe and know — give rise to environments where the principle of co-extensionality might fail. These are called opaque.

### Belief States

(i) John Knows that Scott is Scott.

(ii) Ralph believes that the shortest spy is a spy.
5. Propositional Attitudes

Belief States

(i) John knows that Scott is Scott.
(ii) Ralph believes that the shortest spy is a spy.

States of affairs

(iii) Scott is the author of Waverley.
(iv) The shortest spy is Ortcutt.

Opaque

The speakers holds only certain mental states.
5. Propositional Attitudes

Belief States

(i) John Knows that Scott is Scott.
(ii) Ralph believes that the shortest spy is a spy.

Belief Integration

(i) \(\lnot\) Scott is the author of Waverley.
(ii) \(\lnot\) The shortest spy is Ortcutt.

The speakers holds only certain mental states.
Belief States

(i) John knows that Scott is Scott.
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(iii) Scott is the author of Waverley.
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5. Propositional Attitudes

Belief States

(i) John knows that Scott is Scott.
(ii) Ralph believes that the shortest spy is a spy.

Belief Integration

(i) $\models$ Scott is the author of Waverley.
(ii) $\models$ The shortest spy is Ortcutt.
Champollion's solution...
De Re vs. De Dicto disambiguation

Given that believe combines first with its clausal complement and then with its subject, its type should then be $\langle\langle s,t\rangle,\langle e,t\rangle\rangle$. *(Champollion 2021, p. 506)*

(16) John believes a republican will win.

\[
\begin{align*}
\text{a. } & [\text{Bel(john, } ^\exists x [\text{Repub}(x) \land \text{Win}(x)])] & \text{De Dicto} \\
\text{b. } & \exists x [\text{Repub}(x) \land \text{Bel(john, } ^[\text{Win}(x)])] & \text{De Re}
\end{align*}
\]
When \( m \) is in the scope of the \textit{Bel} operator, its interpretation may vary from \textit{world to world} (De Dicto) but when it is outside (De Re), it just denotes whoever Miss America is in the current world. (p. 507)

(17) John believes miss America is bald.

a. \([\lambda x.\text{Bel}(\text{john}, \ ^\text{Bald}(x))])(m) \quad \text{De Re}

\begin{itemize}
  \item John believes of the person who actually holds the title of Miss America that she is bald.
\end{itemize}
When \( m \) is in the scope of the \textit{Bel} operator, its interpretation may vary from \textit{world to world} (\textit{De Dicto}) but when it is outside (\textit{De Re}), it just denotes whoever Miss America is in the current world. (p. 507)

(17) John believes miss America is bald.

\[
\text{b. } \text{Bel}(\text{john}, \ ^\text{Bald}(m)) \quad \text{De Dicto}
\]

John would assent to the statement “Miss America is bald”.
Crucially, the Ty₂ translation of c does not beta-reduce to that of d. However, the variable is bound in d such that it ranges over of the worlds where “Miss America” is bald.

(17) John believes Miss America is bald.

c. \[\lambda x.\text{Bel}(w,\text{john}(w), \lambda w.\text{Bald}(w,x))](m(w)) \quad \text{De Re}
d. \text{Bel}[w,\text{john}(w), \lambda w.\text{Bald}(w,m(w))] \quad \text{De Dicto}

- w denotes \(w_0\), John(w) denotes always John (\(w_0/w_1\))
- m(w) denotes Camille in \(w_0\) and Victoria in \(w_1\)
- **Problem**: \((m(w))\) is free in c. Instead d’s reading works.
6. Intension and composition
You still use lambdas?

You know there’s Distributional Semantics, right?

Touché Peter
Letting $s$ stand for the type of possible worlds, we now have, for every type $\tau$, a new type $\langle s, \tau \rangle$. The complete type system is now as follows:

- $t$ is a type
- $e$ is a type
- If $\sigma$ and $\tau$ are types, then so is $\langle \sigma, \tau \rangle$
- If $\tau$ is any type, then $\langle s, \tau \rangle$ is a type.

This rule says that for any extensional type you can define you can also add an intensional type which is a function from possible...
Letting $s$ stand for the type of possible worlds, we now have, for every type $\tau$, a new type $\langle s, \tau \rangle$. The complete type system is now as follows:

<table>
<thead>
<tr>
<th>Expression</th>
<th>Example</th>
<th>E-Type</th>
<th>I-Type</th>
</tr>
</thead>
<tbody>
<tr>
<td>Proper name</td>
<td>Luke</td>
<td>$e$</td>
<td>$\langle s, e \rangle$</td>
</tr>
<tr>
<td>Predicate*</td>
<td>Jedi</td>
<td>$\langle e, \tau \rangle$</td>
<td>$\langle s, \langle e, \tau \rangle \rangle$</td>
</tr>
<tr>
<td>Sentence</td>
<td>Luke is a jedi.</td>
<td>$\tau$</td>
<td>$\langle s, \langle \tau \rangle \rangle$</td>
</tr>
</tbody>
</table>

This rule says that for any extensional type you can define you can also add an intensional type which is a function from possible worlds.
6. Intension and composition

Intensional definition of types

\( \alpha \) is an expression of type \( \tau \), then \( \hat{\alpha} \) is an expression of type \( \langle s, \tau \rangle \). Any expression of type \( \langle s, \tau \rangle \) will denote a function from possible worlds to \( D\tau \), where \( D\tau \) is the domain of entities denoted by expressions of type \( \tau \). The official semantic rule for \( \hat{\cdot} \) is as follows:

- If \( \alpha \) is an expression of type \( \tau \), then \( \llbracket \hat{\alpha} \rrbracket_{M,g,w} \) is that function \( f \) with domain \( W \) such that for all \( w \in W : f(w) = \llbracket \alpha \rrbracket_{M,g,w} \)
Example 1

The intension of floats \(^\sim\) \([\text{floats}]^{M,g,w}\) is a function from possible worlds to a function of individuals in truth values (sets of individuals: \(D_s \to D_{et}\)).

\[
\begin{align*}
\text{floats}(w')(\text{yoda}) & : t \\
\text{yoda} & : e \\
\text{floats}(w') & : \langle e, t \rangle \\
w' & : s \\
\text{floats} & : \langle s, \langle e, t \rangle \rangle
\end{align*}
\]

(18) Yoda floats

If not further specified "\(w'\)" stands for the actual world (\(w@\))
Example 2

However, Intensions should be used only when a syntactic-semantic context requires it, rather than always combining everything with intensions. Also, alternatively $\text{floats}(w') = \text{floats}_w$

\[
\text{floats}_w(yoda) : t
\]

\[
yoda : e \quad \text{floats}_w : \langle e, t \rangle
\]
Example 3

Here we see clearly that the extension of “floats” depends on the respective evaluative world. If we abstract based on the world variable \( (\lambda w') \) for the whole sentence \( \langle s, t \rangle \), we get the intension of the sentence, which depends on the intension of floats.

\[
\lambda w'. \text{floats}_{w'}(yoda) : \langle s, t \rangle
\]

\[
\lambda w'
\]

\[
\text{floats}_{w'}(yoda) : t
\]

\[
yoda : e
\]

\[
\text{floats}_{w'} : \langle e, t \rangle
\]

(18) Yoda floats
6. Intension and composition

Logical Space

the intension of a proposition refers to the set of all possible worlds (where this proposition holds). This is representable as the so called \textit{logical spaced} which is divided into two subspaces: \textbf{(1)} the set of worlds which are part of the proposition (or mapped to it) and \textbf{(2)} the set of worlds which are not contained in the proposition.

\[
\left[ \lambda w^* . \text{every}(\text{cat}_{w^*})(\text{sleeps}_{w^*}) \right]
\]
Composition and propositional attitudes

"Believe" is a relation between an individual (attitude holder) and a proposition. If we apply this directly to our analysis, then we can assume that an expression like belief takes a proposition (the object clause) and an individual (the subject) as arguments, resulting in a truth value.

▶️ $\text{believe} \rightarrow \text{believe}_w : \langle\langle s,t\rangle,\langle e,t\rangle\rangle$
Example 4

In the embedded sentence (the complementizer), we first combine the extensional expression \( \text{unicorn}_{w'} \) with its argument \( \text{frodo} \) and obtain an expression of type \( \langle t \rangle \).

\[
\text{believes}_{w'}(\lambda w'. \text{unicorn}_{w'}(\text{frodo})): \langle t \rangle
\]

\[
\text{frodo}: \langle e \rangle, \quad \text{unicorn}_{w'}: \langle e, t \rangle
\]

(19) Gandalf believes Frodo is a unicorn.
Example 4

Because we need the intension of this expression in order to use it as an argument for **believe**, we need the **world variable** \( \lambda w' \) to abstract it. As a result we to obtain an expression of type \( \langle s, t \rangle \).

\[
\text{believes}_{w'} : \langle\langle s,t\rangle,\langle e,t\rangle\rangle \quad \lambda w'.\text{unicorn}_{w'}(\text{frodo}) : \langle s,t \rangle \\
\lambda w' \quad \text{unicorn}_{w'}(\text{frodo}) : \langle t \rangle \\
\text{frodo} : e \quad \text{unicorn}_{w'} : \langle e,t \rangle
\]

(19) Gandalf believes Frodo is a unicorn.
Example 4

Once the complementizer has been combined as argument with the Doxastic operator it needs competitions from the subject.

\[ \text{believes}_w(\lambda w'. \text{unicorn}_w(\text{frodo}))(\text{gandalf}) : \langle t \rangle \]

\[ \text{gandalf} : e \quad \text{believes}_w(\lambda w'. \text{unicorn}_w(\text{frodo})) : \langle e,t \rangle \]

\[ \text{believes}_w : \langle \langle s,t \rangle, \langle e,t \rangle \rangle \quad \lambda w'. \text{unicorn}_w(\text{frodo}) : \langle s,t \rangle \]

\[ \lambda w' \quad \text{unicorn}_w(\text{frodo}) : \langle t \rangle \]

\[ \text{frodo} : e \quad \text{unicorn}_w : \langle e,t \rangle \]

(19) Gandalf believes Frodo is a unicorn.
Example 4

The following tree can be paraphrased like this:

\[
\text{believes}_w(\lambda w'. \text{unicorn}_w(\text{frodo}))(\text{gandalf}) : \langle t \rangle
\]

\[
\text{believes}_w(\lambda w'. \text{unicorn}_w(\text{frodo})) : \langle e, t \rangle
\]

\[
\text{believes}_w : \langle \langle s, t \rangle, \langle e, t \rangle \rangle
\]

\[
\lambda w'. \text{unicorn}_w(\text{frodo}) : \langle t \rangle
\]

\[
\lambda w' \text{ unicorn}_w(\text{frodo}) : \langle t \rangle
\]

\[
frodo : e
\]

\[
\text{unicorn}_w' : \langle e, t \rangle
\]

(19) Gandalf believes Frodo is a unicorn.

[believes, (\lambda w'. \text{unicorn}(\text{frodo}))(\text{gandalf})] = 1, iff Gandalf believes in \( w' \) that Frodo is a unicorn.
Thank you all for the kind attention!
Conclusion

If you need further help or have additional questions, please contact us.