Overview for today

• Recap: Typed lambda calculus
• Montague Grammar
• Functions Applications
• Generalized Quantifies

Reading:
• Coppock, E., and Champollion, L. (2021). Invitation to formal semantics. Manuscript, Boston University and New York University (Ch.6)
Quizz (last week)

Task 1: identify the type of each of the following:

1. Maria
Quizz (last week)

Task 1: identify the type of each of the following:

1. Maria e
Quizz (last week)

Task 1: identify the type of each of the following:
1. Maria e
2. x
Quizz (last week)

Task 1: identify the type of each of the following:

1. Maria \( e \)
2. \( x < e, e > \)
Quizz (last week)

Task 1: identify the type of each of the following:

1. Maria e
2. $x < e, e >$
3. P
Quizz (last week)

Task 1: identify the type of each of the following:

1. Maria $e$
2. $x < e, e >$
3. $P < e, t >$
Quizz (last week)

Task 1: identify the type of each of the following:

1. Maria e
2. x < e, e >
3. P < e, t >
4. P(a)(b)
Quizz (last week)

Task 1: identify the type of each of the following:

1. Maria e
2. x < e, e >
3. P < e, t >
4. P(a)(b) << e, t >, < e, t >>
Quizz (last week)

Task 1: identify the type of each of the following:

1. Maria e
2. x < e, e >
3. P < e, t >
4. P(a)(b) <<< e, t >>, < e, t >>
5. λx.x
Quizz (last week)

Task 1: identify the type of each of the following:

1. Maria e
2. $x < e, e >$
3. $P < e, t >$
4. $P(a)(b) << e, t >, < e, t >>$
5. $\lambda x.x < e, e >$
Quizz (last week)

Task 1: identify the type of each of the following:

1. Maria \( e \)
2. \( x < e, e > \)
3. \( P < e, t > \)
4. \( P(a)(b) << e, t >, < e, t >> \)
5. \( \lambda x.x < e, e > \)
6. \( \lambda f.f \)
Quizz (last week)

Task 1: identify the type of each of the following:

1. Maria \( e \)
2. \( x < e, e > \)
3. \( P < e, t > \)
4. \( P(a)(b) << e, t >, < e, t >> \)
5. \( \lambda x.x < e, e > \)
6. \( \lambda f.f << e, t >, < e, t >> \)
Quizz (last week)

Task 1: identify the type of each of the following:

1. Maria e
2. x < e, e >
3. P < e, t >
4. P(a)(b) << e, t >, < e, t >>
5. λx.x < e, e >
6. λf.f << e, t >, < e, t >>
7. λxλy.R(x, y)
Task 1: identify the type of each of the following:

1. Maria e
2. x < e, e >
3. P < e, t >
4. P(a)(b) << e, t >, < e, t >>
5. λx.x < e, e >
6. λf.f << e, t >, < e, t >>
7. λxλy.R(x, y) << e, < e, t >>
Quizz (last week)

Task 1: identify the type of each of the following:

1. Maria e
2. x < e, e >
3. P < e, t >
4. P(a)(b) << e, t >>, < e, t >>
5. λx.x < e, e >
6. λf.f << e, t >>, < e, t >>
7. λxλy.R(x, y) << e, < e, t >>
8. λxλy.in(x, y)
Quizz (last week)

Task 1: identify the type of each of the following:

1. Maria e
2. x < e, e >
3. P < e, t >
4. P(a)(b) << e, t >, < e, t >>
5. λx.x < e, e >
6. λf.f << e, t >, < e, t >>
7. λxλy.R(x, y) << e, < e, t >>
8. λxλy.in(x, y) << e, < e, t >>
# Types

## Table 3.2: Lexical denotations and their restrictions.

<table>
<thead>
<tr>
<th>Denotation</th>
<th>Type</th>
<th>Restrictions</th>
<th>Category</th>
</tr>
</thead>
<tbody>
<tr>
<td>tina</td>
<td>$e$</td>
<td>-</td>
<td>proper name</td>
</tr>
<tr>
<td>smile</td>
<td>$et$</td>
<td>-</td>
<td>intransitive verb</td>
</tr>
<tr>
<td>praise</td>
<td>$e(et)$</td>
<td>-</td>
<td>transitive verb</td>
</tr>
<tr>
<td>pianist</td>
<td>$et$</td>
<td>-</td>
<td>common noun</td>
</tr>
<tr>
<td>chinese</td>
<td>$et$</td>
<td>-</td>
<td>predicative adjective</td>
</tr>
<tr>
<td>chinese$^\text{mod}$</td>
<td>$(et)(et)$</td>
<td>intersective: $\lambda f_{et}.\lambda x_e.\text{chinese}(x) \land f(x)$</td>
<td>modificational adjective</td>
</tr>
<tr>
<td>skillful$^\text{mod}$</td>
<td>$(et)(et)$</td>
<td>subsective: $\lambda f_{et}.\lambda x_e.\text{(skillful}_{arb}(f))(x) \land f(x)$</td>
<td>modificational adjective</td>
</tr>
<tr>
<td>IS</td>
<td>$(et)(et)$</td>
<td>combinator: $\lambda g_{et}.g$</td>
<td>copula (auxiliary verb)</td>
</tr>
<tr>
<td>A</td>
<td>$(et)(et)$</td>
<td>combinator: $\lambda g_{et}.g$</td>
<td>indefinite article</td>
</tr>
<tr>
<td>HERSELF</td>
<td>$(e(et))(et)$</td>
<td>combinator: $\lambda R_{e(et)}.\lambda x_e. R(x)(x)$</td>
<td>reflexive pronoun</td>
</tr>
<tr>
<td>NOT</td>
<td>$(et)(et)$</td>
<td>logical: $\lambda g_{et}.\lambda x_e. \sim (g(x))$</td>
<td>predicate negation</td>
</tr>
<tr>
<td>AND$^t$</td>
<td>$t(tt)$</td>
<td>logical: $\lambda x_1.\lambda y_1. y \land x$</td>
<td>sentential conjunction</td>
</tr>
<tr>
<td>AND$^e$</td>
<td>$(et)((et)(et))$</td>
<td>logical: $\lambda f_{et}.\lambda g_{et}.\lambda x_e. g(x) \land f(x)$</td>
<td>predicate conjunction</td>
</tr>
</tbody>
</table>

Lambda Calculator: Scratch Tool & exercise hk-chapter6
Quizz (last week)

Task 2: Replace the question mark ‘?’
Lambda Conversion

\[
[\lambda X.\exists x.[P(x) \land X(x)]](\lambda x.\text{Man}(x))
\]

\[
[\lambda X.\exists x.[P(x) \land X(x)]](\lambda y.R(a, y))
\]

\[
[\lambda X.\exists x.[P(x) \land X(x)]](\lambda x.R(a, x))
\]

\[
[\lambda X.\exists x.[P(x) \land X(x)]](\lambda y.R(y, x))
\]

\[
[\lambda X.\forall x[\text{man}(x) \rightarrow X(x)]](\lambda x[\text{mortal}(x)])
\]

\[
[\lambda X\lambda x.\neg X(x)](\lambda x.\text{mortal}(x))
\]

\[
[\lambda X\lambda x.X(x)](\lambda x.\neg\text{mortal}(x))
\]

Lambda Calculator: Scratch Tool & exercise hk-chapter3
Montaque Grammar

Montaque (1930 -1970) semantics or grammar

- English as a formal language
- Universal grammar
- The proper treatment of quantification in ordinary English

Montaque inspiring idea: “I reject the contention that an important theoretical difference exists between formal and natural languages. ” (Montague, 1970)

“Montague grammar is a very elegant and a very simple theory of natural language semantics. Unfortunately its elegance and simplicity are obscured by a needlessly baroque formalization” (Muskens, 1995)
Translation: English Fragments to Formal Representations

What we need:

- a specification of our formal representation language, with syntactic and semantic rules;
- a specification of the syntax of the English expressions we cover;
- a list of lexical entries;
- a list of composition rules.
We have:

- **Typed Lambda Calculus ($L_\lambda$):** the tools semanticists reach for when they want to explore language, state hypotheses, and discuss their ideas.
- **Compositionality:** the meaning of a phrase is a function of the meanings of its immediate syntactic constituents and the way they are combined.

**Lexicon:**

- PN: Lisa
- Neg: not
- V: skateboard

**List of composition rules:** $S \rightarrow NP \ VN \ VP \ NP \rightarrow Det \ N$
Translation: natural language into lambda terms

\[
S \\
PN \quad VP \\
\downarrow \quad \downarrow \\
\text{Lisa} \quad \text{does} \quad \text{Adv} \\
\downarrow \quad \downarrow \\
\text{not} \quad \text{skateboard} \\
\lambda x \neg \text{skateboard}(x)
\]

\[
\neg(\text{skateboard(lisa)}) \\
lisa \\
\lambda x \neg \text{skateboard}(x) \\
\lambda P \neg P(x) \quad \text{skateboard}
\]
Compositionality

Composition Rule 1: Function Application

Let $\gamma$ be a syntax tree whose sub-trees are $\alpha$ and $\beta$ where:
- $\alpha \rightsquigarrow \alpha'$ where $\alpha'$ has type $\langle \sigma, \tau \rangle$
- $\beta \rightsquigarrow \beta'$ where $\beta'$ has type $\langle \sigma \rangle$

then $\gamma \rightsquigarrow \alpha'(\beta')$

```
S
   /
  /  
PN VP
   /
Lisa
   /
  V  PN
  /
loves
  /
Mark
```
Compositionality

Composition Rule 1: Function Application

Let $\gamma$ be a syntax tree whose sub-trees are $\alpha$ and $\beta$ where:
- $\alpha \rightsquigarrow \alpha'$ where $\alpha'$ has type $\langle \sigma, \tau \rangle$
- $\beta \rightsquigarrow \beta'$ where $\beta'$ has type $\langle \sigma \rangle$

then $\gamma \rightsquigarrow \alpha'(\beta')$

$$S$$

```
P N
   VP
```

```
Lisa
   V
     PN
       \langle e, \langle e, t \rangle \rangle
       e
       \lambda y. \lambda x. Loves(x, y)
       loves
       ma
       Mark
```
Composition Rule 1: Function Application

Let $\gamma$ be a syntax tree whose sub-trees are $\alpha$ and $\beta$ where:

- $\alpha \rightsquigarrow \alpha'$ where $\alpha'$ has type $\langle \sigma, \tau \rangle$
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- $\beta \rightsquigarrow \beta'$ where $\beta'$ has type $\langle \sigma \rangle$

then

$\gamma \rightsquigarrow \alpha'(\beta')$

\[
S \\
\text{PN} \\
\text{VP} \\
e \\
l\text{i} \\
\text{Lisa} \\
\lambda x.\text{Loves}(x, \text{ma}) \\
V \\
\langle e, \text{e}, t \rangle \\
\lambda y.\lambda x.\text{Loves}(x, y) \\
\text{loves} \\
\text{PN} \\
e \\
\text{ma} \\
\text{Mark}
\]
Compositionality

Composition Rule 1: Function Application

Let $\gamma$ be a syntax tree whose sub-trees are $\alpha$ and $\beta$ where:
- $\alpha \rightsquigarrow \alpha'$ where $\alpha'$ has type $\langle \sigma, \tau \rangle$
- $\beta \rightsquigarrow \beta'$ where $\beta'$ has type $\langle \sigma \rangle$

then $\gamma \rightsquigarrow \alpha' \left( \beta' \right)$

\[
S \quad t \\
\text{Loves}(li, ma) \\
\text{PN} \quad \text{VP} \\
\text{e} \quad \langle e, t \rangle \\
\text{li} \quad \lambda x.\text{Loves}(x, ma) \\
\text{Lisa} \quad \text{V} \quad \text{PN} \\
\langle e, \langle e, t \rangle \rangle \quad e \\
\lambda y.\lambda x.\text{Loves}(x, y) \quad \text{ma} \\
\text{loves} \quad \text{Mark}
\]
Composition Rule 2: Non-branching Nodes

If \( \beta \) is a tree whose only daughter is \( \alpha \), where \( \alpha \leadsto \alpha' \) then \( \beta \leadsto \alpha' \)

\[ S \]
\[ \text{PN} \quad \text{VP} \]
\[ \text{Lisa} \quad \text{laughs} \]
Compositionality (cont.)

**Composition Rule 2: Non-branching Nodes**

If $\beta$ is a tree whose only daughter is $\alpha$, where $\alpha \rightsquigarrow \alpha'$ then $\beta \rightsquigarrow \alpha'$

- $S$
  - $PN$
  - $VP$
  - Lisa
    - $V$
      - $\langle e, t \rangle$
        - $\lambda x.\text{Laughs}(x)$
          - laughs
Compositionality (cont.)

**Composition Rule 2: Non-branching Nodes**

If $\beta$ is a tree whose only daughter is $\alpha$, where $\alpha \rightsquigarrow \alpha'$ then $\beta \rightsquigarrow \alpha'$

\[
S
\]

\[
P \quad V
\]

\[
\lambda x. \text{laughs}(x)
\]

Lisa

\[
\langle e, t \rangle
\]

\[
\langle e, t \rangle
\]

\[
\lambda x. \text{laughs}(x)
\]

\[
\text{laughs}
\]
Compositionality (cont.)

Composition Rule 2: Non-branching Nodes

If \( \beta \) is a tree whose only daughter is \( \alpha \), where \( \alpha \rightarrow \alpha' \) then \( \beta \rightarrow \alpha' \)

```
S

PN
  e
  li Lisa

VP
  \( \langle e, t \rangle \)
  \( \lambda x.\text{Laughs}(x) \)

V
  \( \langle e, t \rangle \)
  \( \lambda x.\text{Laughs}(x) \)
  laughs
```
Composition Rule 2: Non-branching Nodes

If $\beta$ is a tree whose only daughter is $\alpha$, where $\alpha \leadsto \alpha'$ then $\beta \leadsto \alpha'$

\[
\begin{align*}
S \\
\quad t \\
\quad \text{Laughs}(li) \\
\quad \quad \text{PN} \\
\quad \quad e \\
\quad \quad \text{VP} \\
\quad \quad \langle e, t \rangle \\
\quad \quad \text{li} \\
\quad \quad \lambda x. \text{Laughs}(x) \\
\quad \quad \text{Lisa} \\
\quad \quad \quad \text{V} \\
\quad \quad \quad \langle e, t \rangle \\
\quad \quad \quad \lambda x. \text{Laughs}(x) \\
\quad \quad \quad \text{laughs}
\end{align*}
\]
Transitive verbs:

\[ \langle e, \langle e, t \rangle \rangle \]

\[ \lambda y. \lambda x. P(x, y) \]

Alice loves Connor
Transitive verbs:

\[ \langle e, \langle e, t \rangle \rangle \]

\[ \lambda y. \lambda x. P(x, y) \]

Alice loves Connor
Transitive verbs:

\[ \langle e, \langle e, t \rangle \rangle \]

\[ \lambda y. \lambda x. P(x, y) \]

Alice loves Connor
Transitive verbs:

\[ \langle e, \langle e, t \rangle \rangle \]

\[ \lambda y. \lambda x. P(x, y) \]

Alice loves Connor
Transitive verbs:
\[
\langle e, \langle e, t \rangle \rangle \\
\lambda y. \lambda x. P(x, y)
\]

Alice loves Connor
Transitive verbs:

\[
\langle e, \langle e, t \rangle \rangle \\
\lambda y. \lambda x. P(x, y)
\]

Alice loves Connor
Identity Function

IS:

\( \langle \langle e, t \rangle, \langle e, t \rangle \rangle \)

is \( \rightsquigarrow \lambda P. P \)

Tina is tall

\[ S \]

\[ NP \quad VP \]

Tina

\[ V \quad AP \]

is tall
Identity Function

IS:
\( \langle \langle e, t \rangle, \langle e, t \rangle \rangle \)

is \( \sim \lambda P. P \)

Tina is tall

Diagram:
```
S
 /\  
NP  VP
 /   
V   AP
 /   \ 
\langle e, t \rangle \lambda x. Tall(x)
```
Identity Function

IS:
\[ \langle \langle e, t \rangle, \langle e, t \rangle \rangle \]
is \( \sim \lambda P. P \)
Tina is tall
Identity Function

\[ S \]

\[ \text{IS:} \]
\[ \langle \langle e, t \rangle, \langle e, t \rangle \rangle \]
\[ \text{is} \sim \lambda P. P \]
\[ \text{Tina is tall} \]
Identity Function

IS:
\[ \langle \langle e, t \rangle, \langle e, t \rangle \rangle \]
is \( \rightsquigarrow \lambda P. P \)

Tina is tall
Identity Function

IS: 
\langle \langle e, t \rangle, \langle e, t \rangle \rangle 
is \leadsto \lambda P. P 

Tina is tall
Tina is not tall

\[
S \\
PN \quad VP \\
Tina \\
V \quad NegP \\
is \\
Neg \quad AP \\
not \quad tall
\]
Negation

Tina is not tall

$\text{NegP} \langle e, t \rangle \lambda x. \text{NotP}(x)$

$\text{AP} \langle e, t \rangle \lambda x. \text{Tall}(x)$

$\lambda x. \text{Tall}(x)$

$\text{not}$

$\text{is}$

$\text{V}$

$\text{NegP}$

$\text{VP}$

$\text{PN}$

$S$
Tina is not tall

\[ S \]

\[ \text{PN} \quad \text{VP} \]

\[ \text{Tina} \]

\[ \text{V} \quad \text{NegP} \]

\[ \text{is} \]

\[ \text{Neg} \quad \text{AP} \]

\[ \langle \langle e, t \rangle \langle e, t \rangle \rangle \quad \langle e, t \rangle \]

\[ \lambda P . \lambda x. \neg P(x) \quad \lambda x. \text{Tall}(x) \]

\[ \text{not} \quad \text{tall} \]
Negation

Tina is not tall

S

PN VP

Tina

V NegP

\langle e, t \rangle

\lambda x. \neg Tall(x)

is

Neg

\langle \langle e, t \rangle \langle e, t \rangle \rangle

\lambda P. \lambda x. \neg P(x)

not

AP

\langle e, t \rangle

\lambda x. Tall(x)

tall

not
Tina is not tall

\[
S \\
\text{PN} \quad \text{VP} \\
\text{Tina} \\
\text{V} \\
\langle \langle e, t \rangle \langle e, t \rangle \rangle \\
\lambda P. \ P(x) \\
\text{is} \\
\text{NegP} \\
\langle e, t \rangle \\
\lambda x. \ \neg \text{Tall}(x) \\
\text{Neg} \\
\langle \langle e, t \rangle \langle e, t \rangle \rangle \\
\lambda P. \lambda x. \ \neg P(x) \\
\text{not} \\
\text{AP} \\
\langle e, t \rangle \\
\lambda x. \ \text{Tall}(x) \\
\text{tall}
\]
Negation

Tina is not tall

\[ S \]

\[ \text{PN} \]

\[ \text{VP} \]

\[ \lambda x. \neg \text{Tall}(x) \]

\[ \text{Tina} \]

\[ \text{V} \]

\[ \lambda P. \ P(x) \]

\[ \text{is} \]

\[ \text{NegP} \]

\[ \lambda x. \neg \text{Tall}(x) \]

\[ \text{Neg} \]

\[ \lambda P. \lambda x. \neg P(x) \]

\[ \text{not} \]

\[ \text{AP} \]

\[ \lambda x. \text{Tall}(x) \]

\[ \text{tall} \]
Negation

Tina is not tall

$$S$$

$$\text{PN}$$
- $$e$$
- $$\text{ti}$$
- Tina

$$\text{VP}$$
- $$\langle e, t \rangle$$
- $$\lambda x. \neg \text{Tall}(x)$$

$$\text{V}$$
- $$\langle \langle e, t \rangle \langle e, t \rangle \rangle$$
- $$\lambda P. P(x)$$
- is

$$\text{NegP}$$
- $$\langle e, t \rangle$$
- $$\lambda x. \neg \text{Tall}(x)$$

$$\text{Neg}$$
- $$\langle \langle e, t \rangle \langle e, t \rangle \rangle$$
- $$\lambda P. \lambda x. \neg P(x)$$
- not

$$\text{AP}$$
- $$\langle e, t \rangle$$
- $$\lambda x. \text{Tall}(x)$$
- tall
Tina is not tall
Indefinite Article

A:
\[
\langle \langle e, t \rangle, \langle e, t \rangle \rangle \\
\lambda P. P
\]
Tina is an athlete.

Tina

is

an athlete
Indefinite Article

A:
\[ \langle \langle e, t \rangle, \langle e, t \rangle \rangle \]
\[ a \sim \lambda P.P \]

Tina is an athlete.
Indefinite Article

A:
\[ \langle \langle e, t \rangle, \langle e, t \rangle \rangle \]
\[ a \sim \lambda P. P \]
Tina is an athlete.
Indefinite Article

A: \[ \langle \langle e, t \rangle, \langle e, t \rangle \rangle \]
\[ a \leadsto \lambda P.P \]
Tina is an athlete.
Indefinite Article

A: \[
\langle \langle e, t \rangle, \langle e, t \rangle \rangle \\
a \leadsto \lambda P.P \\
\]

Tina is an athlete.

\[
\text{Tina} \\
\text{is} \\
\lambda x.\text{Athlete}(x) \\
\lambda P.P \\
\text{an} \\
\lambda x.\text{Athlete}(x) \\
\]

\[
\text{NP} \\
\text{VP} \\
\text{S} \\
\]

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A: 
\[ \langle \langle e, t \rangle, \langle e, t \rangle \rangle \]
\[ a \sim \lambda P. P \]
Tina is an athlete.
Indefinite Article

A:
\( \langle \langle e, t \rangle, \langle e, t \rangle \rangle \)
\( a \sim \lambda P. P \)
Tina is an athlete.
A: $\langle\langle e, t\rangle, \langle e, t\rangle\rangle \rightarrow \lambda P. P$

Tina is an athlete.
Adjectives of type $\langle e, \langle e, t \rangle \rangle$

$a \sim \lambda x. x$

Hannah is fond of Gabriel.
Prepositional Phrases

Adjectives of type $\langle e, \langle e, t \rangle \rangle$

$a \rightsquigarrow \lambda x.x$

Hannah is fond of Gabriel.
Prepositional Phrases

Adjectives of type \( \langle e, \langle e, t \rangle \rangle \)

\( a \mapsto \lambda x.x \)

Hannah is fond of Gabriel.
Prepositional Phrases

Adjectives of type $\langle e, \langle e, t \rangle \rangle$

$a \mapsto \lambda x.x$

Hannah is fond of Gabriel.
Prepositional Phrases

Adjectives of type \( \langle e, \langle e, t \rangle \rangle \)
\( a \mapsto \lambda x.x \)
Hannah is fond of Gabriel.
Adjectives of type $\langle e, \langle e, t \rangle \rangle$

$a \leadsto \lambda x.x$

Hannah is fond of Gabriel.
Adjectives of type $\langle e, \langle e, t \rangle \rangle$

$\begin{align*}
a & \mapsto \lambda x. x \\
\text{Hannah is fond of Gabriel.}
\end{align*}$
Adjectives of type \( \langle e, \langle e, t \rangle \rangle \)

\( a \leadsto \lambda x.x \)

Hannah is fond of Gabriel.
Prepositional Phrases

Adjectives of type $\langle e, \langle e, t \rangle \rangle$

$a \mapsto \lambda x.x$

Hannah is fond of Gabriel.
Prepositional Phrases

Adjectives of type \( \langle e, \langle e, t \rangle \rangle \)

\( a \leadsto \lambda x.x \)

Hannah is fond of Gabriel.
Adjectives of type $\langle e, \langle e, t \rangle \rangle$

$a \rightsquigarrow \lambda x.x$

Hannah is fond of Gabriel.
Quantification: type $\langle\langle e, t \rangle, t \rangle$

*Everybody, everything, somebody, something, nobody and nothing*

**Example**

*Everybody dances.*

\[
everybody \leadsto \lambda P. \forall x. P(x) \langle\langle e, t \rangle, t \rangle
\]

\[
something \leadsto \lambda P. \exists x. P(x) \langle\langle e, t \rangle, t \rangle
\]

\[
nobody \leadsto \lambda P. \neg \exists x. P(x) \langle\langle e, t \rangle, t \rangle
\]

\[
\forall x. \text{Dances}(x)
\]

\[
t
\]

\[
\lambda P. \forall x. P(x) \langle\langle e, t \rangle, t \rangle
\]

\[
\lambda x. \text{Dances}(x) \langle e, t \rangle
\]

\[
everybody
\]

\[
dances
\]
Quantification: why not $e$

no composition rule to combine two expressions of type $\langle e, t \rangle$. Similarly: somebody/everybody/nobody is brave...

\[
S
\]

\[
S
\]

\[
S
\]

\[
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\]
Quantification: why not $e$ (cont.)

$e$ should validate subset-to-superset inference, e.g.

<table>
<thead>
<tr>
<th>Subset-to-superset inference</th>
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</thead>
<tbody>
<tr>
<td>Susan came yesterday morning.</td>
</tr>
<tr>
<td>$\therefore$ Susan came yesterday.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Law of non-contradiction</th>
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</thead>
<tbody>
<tr>
<td>$e$ do not always adhere to $[p \land \neg p]$ false for any $p$</td>
</tr>
<tr>
<td>Mont Blanc is higher than 4,000m, and Mont Blanc is not higher than 4,000m. $[p \land \neg p] \vdash \bot$</td>
</tr>
<tr>
<td>More than two mountains are higher than 4,000m, and more than two mountains are not higher than 4,000m. $[p \land \neg p] \not\vdash \bot$</td>
</tr>
</tbody>
</table>
Quantification: predicates of predicates

*Every, some and no*

### Example

**Every cat meows.**

\[
\text{every} \mapsto \lambda P . \lambda P'. \forall x. [P(x) \rightarrow P'(x)] \quad \langle \langle e, t \rangle, \langle \langle e, t \rangle, t \rangle \rangle
\]

\[
\text{some} \mapsto \lambda P . \lambda P'. \exists x. [P(x) \land P'(x)] \quad \langle \langle e, t \rangle, \langle \langle e, t \rangle, t \rangle \rangle
\]

\[
\text{nobody} \mapsto \lambda P . \lambda P'. \neg \exists x. [P(x) \land P'(x)] \quad \langle \langle e, t \rangle, \langle \langle e, t \rangle, t \rangle \rangle
\]
Quantification: type \( \langle \langle e, t \rangle, \langle \langle e, t \rangle, t \rangle \rangle \) (cont.)

\[
S \\
\lambda P'. \forall x.[Cat(x) \rightarrow P'(x)] \\
\lambda x. Meows(x) \\
\langle e, t \rangle \\

\[
\forall x.[Cat(x) \rightarrow Meows(x)] \\
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Generalised Quantifiers

There are many more quantifiers: few, exactly two, more than five sonatas, most of the...
Generalised Quantifiers

There are many more quantifiers: few, exactly two, more than five sonatas, most of the quantification over time and space such as always, sometimes, never, annually, everywhere, somewhere and nowhere;

Example

the generalized quantifier every boy denotes the set of sets of which every boy is a member:

\{
X \mid \forall x (\text{Boy}(x) \rightarrow x \in X)\}
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There are many more quantifiers: few, exactly two, more than five sonatas, most of the quantification over time and space such as always, sometimes, never, annually, everywhere, somewhere and nowhere; expressions such as twice (as in I called you twice and more than five times, or twice every day in I will call you twice every day and many more.
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\[ \{ X \mid \forall x (Boy(x) \rightarrow x \in X) \} \]
In formal logic, if \( p \) is a formula that denotes a proposition then the expressions \( \forall x.p \) and \( \exists y.p \) are quantifications, saying that \( p \) is true of all individual objects and that \( p \) is true of at least one such object, respectively.
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Such quantifications, which range over all individual objects in a universe of discourse, cannot be expressed in natural languages. It just is not possible to say that something is true “for all” or “for some”, where “all” and “some” would refer to any conceivable object.
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Noun phrases (NPs), expressing (generalized) quantifiers in natural language, typically consist of two parts: (1) a noun, in grammatical analysis called the ‘head’ of the NP, possibly with one or more adjectives, prepositional phrases or other modifiers, and (2) one or more determiners such as “a”, “the”, “all”, “some”, “most”, “half of the”, and “less than 200”.

Generalised Quantifiers (cont.)

The head noun with its modifiers is called the **restrictor** of the quantifier and indicates a certain domain that the quantifier ranges over. The term **source domain** is used to indicate the set of entities (or, alternatively, the property that characterises these entities) that the restrictor refers to.

Example

Everybody must hand in his essay before Thursday next week.
The proposal was accepted by all the twenty-seven member countries.

Westerstahl (1985) introduced the term '**context set**' to designate the contextually determined subset of a source domain that is relevant in a quantified predication.
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The presence of a restrictor component forms the fundamental difference between quantification in logic and quantification in natural language: quantification in logic is always understood as ranging over the set of all entities in a given universe of discourse, whereas quantification in natural language is restricted to a source domain that is made explicit in the quantifier’s restrictor.

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Generalised Quantifiers (cont.)

The determiner part may be a sequence of determiners of different types, distinguished by sequencing and co-occurrence restrictions. For example, in English grammar it is customary the make a distinction between:

- **predeterminers** express the (absolute or proportional) quantitative involvement of the source domain, and may, in addition, say something about the distribution of a quantifying predicate over the source domain;
- **central determiners** express the definiteness of the NP;
- **postdeterminers** express a proposition about the cardinality of the reference domain.

**Example**

All of her nine grandchildren are boys.
Some quantifiers are FOL definable.

Example: exactly two things

\[ \lambda P. \exists x. \exists y. \neg (x = y) \land P(x) \land P(y) \land \neg \exists z. P(z) \land \neg (z = x) \land \neg (z = y) \]

Others are not FOL definable

Example: most swans = \{ \[ P \subseteq D \} : |SWAN \cap P| > |SWAN - P| \}

Most swans = \{ \[ P \subseteq D \} : |P| > |D - P| \}

One of the three cats = \{ \[ P \subseteq D \} : |P \cup CAT| / |CAT| \geq 1 / 3 \}

Solution: relate and compare sets; represent in second-order logics.

Example:

\[ \exists X [ |X| = 2 \land \forall x [x \in X \rightarrow \text{Man}(x)] \land \exists y [Piano(y) \land \text{Carry}(x, y)]] \]

Two men carried a piano upstairs.
Generalised Quantifiers (cont.)

Some quantifiers are FOL definable.

Example
exactly two things ⇝ \( \lambda P \cdot \exists x \cdot \exists y \cdot \neg (x = y) \land P(x) \land P(y) \land \neg \exists z \cdot P(z) \land \neg (z = x) \land \neg (z = y) \)

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Example
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**Generalised Quantifiers (cont.)**

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most swans $\overset{\text{def}}{=} \{ P \subseteq D_e : |\text{SWAN} \cap P| > |\text{SWAN} - P| \}$

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**Example**

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Generalised Quantifiers (cont.)

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exactly two things $\iff \lambda P. \exists x. \exists y. \neg(x = y) \land P(x) \land P(y) \land \neg \exists z. P(z) \land \neg(z = x) \land \neg(z = y)$

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Example

all of the crew escaped the blast $\rightsquigarrow \forall x[C(x) \implies E(x)]$ or $=_{\text{def}} C \subseteq E$

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all of the crew escaped the blast \( \sim \forall x [C(x) \implies E(x)] \) or \( =_{\text{def}} C \subseteq E \)

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seems universal across all human languages! Is SEMANTIC UNIVERSAL property
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All quantifiers seem to have the property of CONSERVATIVITY: quantifiers only care about the elements in the first set they combine with, and so they ignore anything in the second set that’s not already in the first.

\([Q](A)(B) \leftrightarrow [Q](A)(A \cap B)\)
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Quantifiers that do not depend on $|E \cup C|$ satisfy EXTENSION, e.g. only, there constructions
Quantifiers in object position

Exercises: hk-chapter6 & example 3
Quizz for Today

TBA