Exercise 3

0.1 Probability rules

- Suppose that we have three coloured boxes \( r \) (red), \( b \) (blue), and \( g \) (green). Box \( r \) contains 3 apples, 4 oranges, and 3 limes, box \( b \) contains 1 apple, 1 orange, and 0 limes, and box \( g \) contains 3 apples, 3 oranges, and 4 limes. If a box is chosen at random with probabilities \( p(r) = 0.2 \), \( p(b) = 0.2 \), \( p(g) = 0.6 \), and a piece of fruit is removed from the box (with equal probability of selecting any of the items in the box), then what is the probability of selecting an apple? If we observe that the selected fruit is in fact an orange, what is the probability that it came from the green box?

- Scenario 1: I flip two fair coins. You know that I got at least one head. What is the probability that I got two heads?

- Scenario 2: I flip two fair coins. You know that I got at least one head and it was a euro from Malta. First assume that one in two of my coins are euros from Malta. What is the probability that I got two heads? What is this probability in the general case when the proportion of Malta euros is \( m \)?

0.2 Expectations

Suppose that the two discrete random variables \( X \) and \( Z \) are statistically independent. Show that the mean and variance of their sum satisfies:

\[
E[X + Z] = E[X] + E[Z]
\]

\[
var[X + Z] = var[X] + var[Z]
\]

Please send your solutions by Tuesday Nov 20 to: psr-tutorial@lsv.uni-saarland.de

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