1. For the example on the board, give three shortest path trees other than that shown.

2. Let $G = (V, E)$ be a weighted, directed graph that contains no negative-weight cycles. Let $s \in V$ be the source vertex, and let $G$ be initialized by the Initialize-Single-Source($G, s$) procedure defined in class. Prove that there exists a sequence of $|V| - 1$ relaxation steps that produces $d[v] = \delta(s, v)$ for all $v \in V$.

Solution: Because it contains no cycles with negative weights, the Bellman-Ford algorithm will terminate on $G$ after assigning the shortest path distances to all $v \in V$ and correctly setting the predecessor of each $v \in V$. The resulting shortest path tree $G'$ represents a partial ordering of all $v \in V$. To obtain a linear ordering, a topological sort can be performed on $G'$. The topological sort will provide the desired result because $G'$ is a tree and hence acyclic. Thereafter, the nodes can be relaxed in the order specified by the topo sort. There are at most $|V| - 1$ such relaxation steps, which occurs when $G'$ has a single leaf node. It must then hold that $d[v] = \delta(s, v)$ for all $v \in V$, because each $v$ has been relaxed along one of its shortest paths.

3. Arbitrage is the use of discrepancies in currency exchange rates to transform one unit of currency into more than one unit of the same currency. For example, suppose that 1 U.S. dollar buys 46.4 Indian rupees, 1 Indian rupee buys 2.5 Japanese yen, and 1 Japanese yen buys 0.0091 U.S. dollars. Then by converting currencies, a trader can start with 1 U.S. dollar and