12. Wiener Filter
Noise Removal

- Noise is present in many situations of daily life
- Microphones will record noise and speech
- Goal:
  - Reconstruct original speech signal
Filtering of a Signal to Remove Noise

Original signal \( s[k] \) through a filter \( h[k] \) results in a filtered signal \( \tilde{s}[k] \). The input includes noise \( n[k] \).
Idea of Wiener Filter

• Minimize Euclidian distance between the original signal and the filtered signal

\[ e^2 = E[k] - s[k] \]

\[ E \] expectation value of the stochastic process

Nach: Norbert Wiener (November 26, 1894 – March 18, 1964)
Solution in the Time Domain

• See black board
Wiener Hopf Equation

\[ h[i] * (\varphi_{ss}[i] + \varphi_{nn}[i]) = \varphi_{ss}[i] \]
Toeplitz Matrix and Levinson-Durbin Recursion

- Toeplitz matrix
- Levinson (Durbin) recursion
Symmetry of Impulse Response

As

\[
\begin{pmatrix}
\varphi_{ss}[-N] \\
\vdots \\
\varphi_{ss}[N]
\end{pmatrix}
= 
\begin{pmatrix}
\varphi_{ss}[N] \\
\vdots \\
\varphi_{ss}[-N]
\end{pmatrix}
\]

and the Topelitz matrix is symmetric

\[
\Rightarrow 
\begin{pmatrix}
h[-N] \\
\vdots \\
h[N]
\end{pmatrix}
= 
\begin{pmatrix}
h[N] \\
\vdots \\
h[-N]
\end{pmatrix}
\]

\[h[i] = h[-i]\]

symmetry of the Wiener filter
Frequency Solution of Wiener-Hopf Equation

\[ H(\omega) \Phi_{ss}(\omega) + \Phi_{nn}(\omega) = \Phi_{ss}(\omega) \]

\[ H(\omega) = \frac{\Phi_{ss}(\omega)}{\Phi_{ss}(\omega) + \Phi_{nn}(\omega)} \]

Can you explain what it does to specific frequencies?
Wiener Filter in Frequency Domain

\[ H(\omega) = \frac{\Phi_{ss}(\omega)}{\Phi_{ss}(\omega) + \Phi_{nn}(\omega)} \]

- Suppresses frequencies where noise is present
- Other frequencies remain unchanged
Heuristic Modification

\[ H_\eta(\omega) = \left( \frac{\Phi_{ss}(\omega)}{\Phi_{ss}(\omega) + \Phi_{nn}(\omega)} \right)^\eta \]

- Tune how “strong” the filter should be
Estimating the Noise AKF

- Use pauses, where there is no signal
- Voice activity detection (VAD)
Application to Images

Original

Distorted (blurred)

Reconstructed (using Wiener-F)

Wiener filtering

A signal, which is transmitted from the sender to the receiver is often impaired by various forms of distortions. Wiener filtering is a method to recover the original signal as close as possible from the received signal. Consider the following situation: An original signal $s(t)$ is transmitted through an information channel (cable, wireless channel, storage medium). The received signal $x(t)$ is impaired by two different effects. Firstly, the channel may not have a perfect "delta-function" response so that the original signal $s(t)$ is convolved with some known impulse response $g(t)$ to give a smeared signal $v(t) = g(t) * s(t)$. Secondly, noise $n(t)$ may be added to $v(t)$ to give finally the signal $x(t) = v(t) + n(t)$ at the receiver. Our task is to find the optimal filter (or Wiener filter) $h(t)$ which, when applied to the signal $x(t)$ produces a signal $y(t)$ that is as close as possible to the uncorrupted signal $s(t)$. In other words we want to estimate the true signal $s(t)$.

The frequency response of the Wiener filter is given by:

$$H(jw) = \frac{\phi_{ss}(jw) G^*(jw)}{\phi_{ss}(jw) |G(jw)|^2 + \phi_{nn}(jw)}$$

(see the textbook at page 461ff.)
\[ H(jw) = \frac{\phi_{ss}(jw) | G(jw) |^2 + \phi_{nn}(jw)}{\phi_{ss}(jw) | G(jw) |^2} \]
**Wiener Filter**

**I am interested in collaboration.**

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**Code Metrics:**  Full report

**What is this?**

**Author:** Esfandiar Zavarehei

**Summary:** Wiener Filter for Noisy Speech Enhancement

**MATLAB Release:** R14SP1

**Description:** Wiener Filter for Noisy Speech Enhancement, implemented as proposed by Scalart 1996. Uses a decision directed method to track the a priori SNR. Note that the first 0.25 sec of the signal is used to model the noise, that is, it is assumed that speech starts after that 0.25 sec

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Summary

- Wiener filter
  - Minimize quadratic distance to original signal
- Wiener-Hopf equation
- Time domain solution
- Frequency domains solution