Exercise 1

The aim of this exercise is to implement and use an LDA.

(6P) Subtask 1.1

There are two matrices given: \( S_w = \begin{pmatrix} a & b \\ b & a \end{pmatrix} \) and \( S_b = \begin{pmatrix} c & d \\ d & c \end{pmatrix} \) with \( c^2 \neq d^2 \) and \( d \neq 0 \). Compute the generalized Eigenvalues \( \lambda_1, \lambda_2 \), which solve the equations

\[
S_w \phi = \lambda_i S_b \phi, \quad i = 1, 2
\]

of the generalized Eigenvalue Problem for all generalized Eigenvectors \( \phi \in \mathbb{R}^2 \). What happens for \( d = 0 \) (, \( d = 0 \) and \( c = 1 \))?

(2P) Subtask 1.2

Generate two 2-dimensional, normal distributed datasets \( C_1, C_2 \) with 1000 data points for the two classes \( C_1 \) and \( C_2 \). For \( C_1 \), use a mean of \( \mu_x = 3, \mu_y = 3 \) and a variance of \( \sigma^2_x = 0.1, \sigma^2_y = 1 \). For \( C_2 \), use a mean of \( \mu_x = -3, \mu_y = -3 \) and a variance of \( \sigma^2_x = 1, \sigma^2_y = 0.1 \).

(4P) Subtask 1.3

Implement an LDA \(^2\) and use the dataset of subtask 1.2) in order to visualise the functionality of the LDA in form of a scatter plot.

---

\(^1\)You can use \texttt{randn} in Matlab. The data can be shown with \texttt{scatter}.

\(^2\)You can use the Matlab function \texttt{cov}, \texttt{eig} and \texttt{sort}